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FUZZY INFERENCE SYSTEMS BASE ON POLYNOMIAL CONSEQUENTS OF FUZZY RULES

Abstract: Various fuzzy inference systems that operate on the basis of polynomial consequents of fuzzy rules. As well as inference methods for such systems, in particular, Takagi-Sugeno fuzzy inference systems, their differences from other popular fuzzy systems, such as Mamdani systems, etc., are considered. The attention is focused on the features of the functioning of such systems both in the construction of elementary fuzzy systems. The Systems for which the calculation of the general logical conclusion involves intermediate levels of logical inference with many hierarchically interconnected blocks of fuzzy rules. Fuzzy sets of type 2 are considered, the membership index of which is a fuzzy term of the first type. This allows you to take into account the secondary fuzziness of linguistic concepts in the design of intelligent systems based on fuzzy inference. Fuzzy systems of the second type based on Takagi-Sugeno systems and the iterative Karnik-Mendel algorithm are considered to obtain a logical conclusion for fuzzy systems with the interval membership functions of the second type in the antecedents of fuzzy rules. The proposed procedure for lowering the order of fuzzy rules for higher-order Takagi-Sugeno fuzzy systems is described and justified. A fuzzy inference method for higher-order fuzzy systems based on the partition of a set of input variables is proposed. It is proposed to build a separate block of fuzzy rules for each of the input subspaces in the presence of a common polynomial. Which is a higher-order consequent, that reduces the total number of fuzzy rules in blocks.

Keywords: fuzzy system, consequent of the fuzzy rule, higher-order fuzzy systems, fuzzy inference.

Introduction

Since the creation of the Lotfie Zadeh apparatus of fuzzy logic [1], it has become possible to operate with qualitative (categorical) quantities that are linguistic in nature [2]. The appearance of the first fuzzy inference algorithm laid the foundation for the existence of a whole class of systems [3]. Fuzzy logical inference (FLI) is often used in the development of intelligent systems that operate on the basis of predicate rule blocks. If necessary, take into account the fuzzy (vague) nature of the quantities being operated, often arising also when using linguistic variables [2, 4, 5]. When constructing fuzzy systems [6], an expert in the problem area plays an important role. The knowledge and experience of whom are used primarily in constructing blocks of fuzzy rules, due to insufficient knowledge in solving a narrow circle of expert and intellectual problems [7–8]. Using fuzzy inference algorithms, expert tasks [9], clustering problems [10-11], selection, optimization, decision making, pattern recognition [7, 12-15], etc. are solved.

In general, we can say that fuzzy logical systems (FLS) are a kind of (intelligent) input to output converters based on a system of rules [3, 12, 14]. The nature of the fuzziness of such

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systems may consist in operating with fuzzy input data, fuzzy output data, and fuzziness can be exclusively inside the system itself [8]. In last case, a fuzzy system can exist both in a fuzzy environment, and in an environment of clear quantities.

There are many algorithms for fuzzy inference [3, 15–17], such as Mamdani, Tsukamoto, Larsen, Takagi-Sugeno. The first three of them have common features, namely, the fuzzy nature of the consequents of the rules and the need to use defuzzification mechanisms on one or the other stage of the algorithm. A characteristic feature of the fuzzy inference of Takagi-Sugeno has that fuzziness is stored only in the antecedents of the rules. While the consequents of the rules are presented in the form of functional (polynomial) dependencies on clear (not fuzzy) input parameters [8, 15]. This way, the fuzzy Takagi-Sugeno systems produce clear output value. The input values must also be clear numbers with their subsequent fuzzification inside the fuzzy system.

Systems with polynomial consequents of fuzzy rules have a number of advantages, such as the relative simplicity of their learning, the absence of no valuable operations to defuse output values [7, 12]. At the same time, when constructing hierarchical blocks of Takagi-Sugeno fuzzy rules [9, 18–21], the problem of accumulation of fuzziness inherent in fuzzy systems like Mamdani is absent. Based on the information that was given before, this paper discusses the features of functioning, as well as the further development of fuzzy systems. Based on Takagi-Sugeno systems with polynomial consequents of fuzzy rules. A method of fuzzy inference for higher-order Takagi-Sugeno systems is developed, as well as a procedure for lowering the order of fuzzy rules for such systems.

The purpose and objectives

The aim of this work is to study fuzzy inference systems that operate on the basis of blocks of fuzzy rules with polynomial consequents. During the research, the following tasks were set:

consider fuzzy Takagi-Sugeno systems of the first and zero orders, as well as fuzzy inference systems of type 2 based on rules of the Takagi-Sugeno type, and fuzzy inference algorithms for such systems;

develop a fuzzy inference method based on higher-order Takagi-Sugeno rule blocks with a procedure for lowering the order of fuzzy rules.

Takagi-Sugeno Fuzzy Inference

Takagi-Sugeno fuzzy inference systems consist of blocks of fuzzy predicate rules *if-then* [8, 15]. Each rule of the ordinary Takagi-Sugeno system (zero order) contains polynomials of degree zero as a consequent of the rule:

$$IF\ x_1\ is\ A_1^l\ AND\ x_2\ is\ A_2^l\ AND\ \ldots\ AND\ x_m\ is\ A_m^l\ THEN\ \ y^l = a_0^l, \tag{1}$$

where $x_1, x_2, \ldots, x_m$ – are the values of the input variables represented by fuzzy sets; $A_1^l, A_2^l, \ldots, A_m^l$ – fuzzy sets in the antecedent of rule *l*; *l* – rule number; a_0^l – the initial value for the consequent of the *l*-th rule, which is presented as a constant.

The Takagi-Sugeno first-order fuzzy inference system is based on blocks of fuzzy predicate rules IF-THEN of the following form:

$$\begin{aligned} &R_j : IF\ x_1\ is\ \ A_{1j}\ AND\ x_2\ is\ \ A_{2j}\ AND\ \ldots\ AND\ x_n\ is\ \ A_{nj} \\ &THEN\ \ y_j = g_j(x_1, x_2, \ldots, x_n), \quad j = 1, 2, \ldots, N \end{aligned} \tag{2}$$

where is the function of finding the sub-rule: $g_j(x_1, x_2, \ldots, x_n) = \omega_0 + \omega_1 x_1 + \omega_2 x_2 + \ldots + \omega_n x_n$ is a linear functional dependence on the clear values of the input premises of the fuzzy system, *N* is the total number of rules within the block.

The general conclusion of a fuzzy system like Takagi-Sugeno is calculated according to (3):

$$y = \frac{\sum_{j=1}^{N} g_j \mathop{T}_{i=1}^{m_j} \mu_{ij}(x_i)}{\sum_{j=1}^{N} \cdot \mathop{T}_{i=1}^{m_j} \mu_{ij}(x_i)} \tag{3}$$

where $\mu_{ij}(x_i)$ is a function of the input parcels belonging to a fuzzy term, T – is a T-norm, where the operation of finding the minimum is usually used as a conjunction.

Takagi-Sugeno Fuzzy Inference Type 2

Fuzzy Takagi-Sugeno systems were further developed with the advent of T2 FS [22–23] (fuzzy sets of type 2 (T2) proposed by L. Zadeh [2]. With the appearance of the Karnik-Mendel algorithm [23–24] in fuzzy inference systems Takagi-Sugeno made it possible to use T2 FS (in particular, interval ones) in the antecedents of fuzzy rules. Unlike fuzzy systems of type 2 with Mamdani-type subclauses, the Karnik-Mendel algorithm for Takagi-Sugeno systems has (under certain conditions) slightly less computational complexity for fulfilling reduction operations [22].

A fuzzy set of type n, $n = 2, 3 \ldots$ is a fuzzy set whose membership function value is a fuzzy set of type n – 1 [23–24]:

$$\widetilde{A} = \int_{X_N} \int_{X_{N-1}} \cdots \int_{X_1} \frac{\mu_{\widetilde{A}}(X_1,\ldots,X_N)}{(X_1,\ldots,X_N)} \tag{4}$$

Therefore, a continuous fuzzy set of the second type can be represented as:

$$\widetilde{A} = \int_X \frac{\mu_{\widetilde{A}}(x)}{x} = \int_X \frac{\left[\int_{J_x^u} \frac{f_x(u)}{u}\right]}{x}, \quad J_x^u = \left\{(x,u) : u \in \left[\overline{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{A}}(x)\right]\right\} \subseteq [0,1] \tag{5}$$

A discrete fuzzy set of type 2 can be represented respectively:

$$\widetilde{A} = \sum_{j=1}^{N} \frac{\mu_{\widetilde{A}}(x_j)}{x_j} = \sum_{j=1}^{N} \frac{\left[\sum_{i=1}^{M_j} \frac{f_x(u_i)}{u_i}\right]}{x_j}, \quad u_i \in J_x^u \subseteq u \in \left[\overline{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{A}}(x)\right] \subseteq [0,1],\ x_j \in X \tag{6}$$

where f_x is a secondary membership function, $\overline{\mu}_{\widetilde{A}}(x)$ is a upper bound on the value of the primary membership function:

$$\overline{\mu}_{\widetilde{A}}(x) = \sup J_x^u, \qquad x \in X \tag{7}$$

$\underline{\mu}_{\widetilde{A}}(x)$ is a lower bound on the value of the primary membership function:

$$\underline{\mu}_{\widetilde{A}}(x) = \inf J_x^u, \qquad x \in X \tag{8}$$

Figure 1 shows graphically T2 FS using triangular functions for the primary and secondary membership functions.

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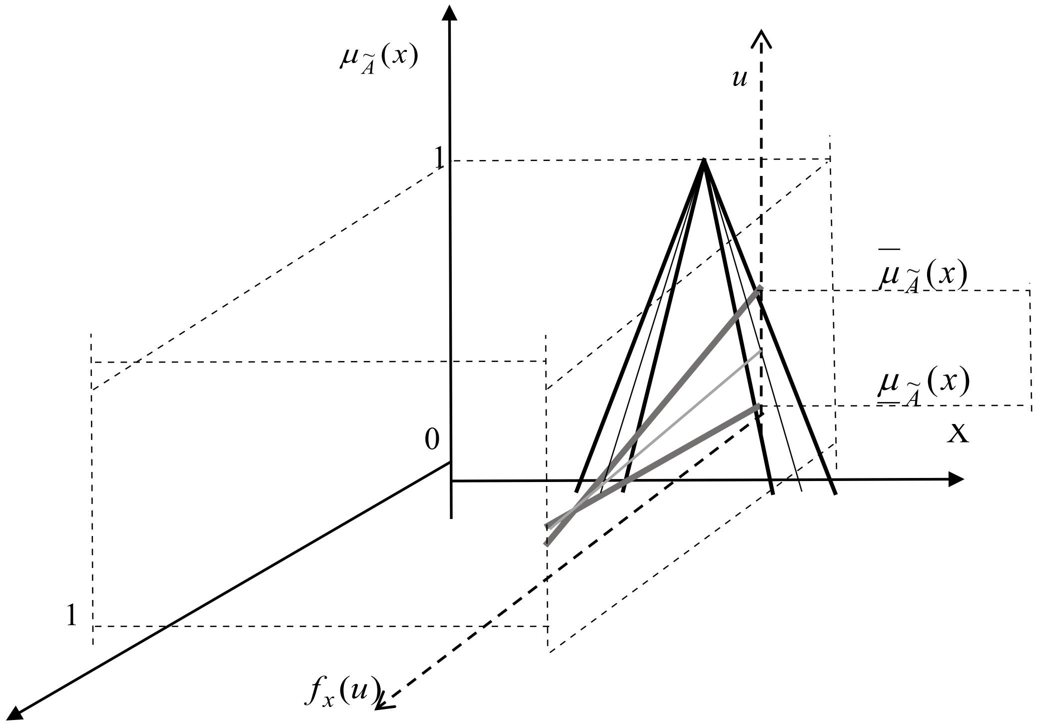

Fig. 1. Graphical representation of fuzzy sets of type 2

The Karnik-Mendel algorithm for fuzzy Takagi-Sugeno type 2 systems with interval secondary membership functions.

In this case, Takagi Sugeno FLS T2 involve the use of interval type 2 (IT2) fuzzy sets (FS) [22] in the antecedents of fuzzy rules of the form:

$$\begin{aligned} &R^k : IF\ x_1\ is\ \tilde{A}_1^k\ AND\ ...\ AND\ x_m\ is\ \tilde{A}_m^k \\ &THEN\ g^k(x) = w_0^k + w_1^k x_1 + \ldots w_m^k x_m \end{aligned} \tag{9}$$

where $\tilde{A}_1^k \ldots \tilde{A}_m^k$ – IT2 FS, k is the number of the rule. IT2 FS have the form:

$$\tilde{A} = \int_X \frac{\mu_{\tilde{A}}(x)}{x} = \int_X \frac{\left[\int_{J_x^u} \frac{1}{u}\right]}{x}, \qquad J_x^u = \left\{(x,u) : u \in \left[\overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x)\right]\right\} \subseteq [0,1] \tag{10}$$

According to the Karnik-Mendel fuzzy inference algorithm [22, 24], the operations of finding the values of the sub-rules of the rule base $(g^k(x), k = 1\ldots N)$, as well as calculating the intervals $\left[\underline{f}^k(x), \overline{f}^k(x)\right]$ for each rule.

This is followed by an iterative operation of decreasing the type and finding the interval output [22], which are determined according to (11–13) (a research of the adequacy of interval fuzzy models of type 2 is presented in [25]).

$$G(\mathrm{x}) = \left[g_l(\mathrm{x}), g_r(\mathrm{x})\right] \tag{11}$$

$$g_l(\mathrm{x}) = \min_{\substack{f_j^k(\mathrm{x}) \in \left[\underline{f}^k(\mathrm{x}), \overline{f}^k(\mathrm{x})\right] \\ k=1,\ldots,N}} \frac{\sum_{k=1}^{N} f_j^k(\mathrm{x}) g^k(\mathrm{x})}{\sum_{k=1}^{N} f_j^k(\mathrm{x})} \quad (12)$$

$$g_r(\mathrm{x}) = \max_{\substack{f_j^k(\mathrm{x}) \in \left[\underline{f}^k(\mathrm{x}), \overline{f}^k(\mathrm{x})\right] \\ k=1,\ldots,N}} \frac{\sum_{k=1}^{N} f_j^k(\mathrm{x}) g^k(\mathrm{x})}{\sum_{k=1}^{N} f_j^k(\mathrm{x})} \quad (13)$$

It is worth mentioning that the polynomials of consequents of fuzzy rules are calculated many times over the entire interval $\left[\underline{f}^k(x), \overline{f}^k(x)\right]$, since the numbers obtained may differ.

A clear conclusion of the FLS T2 is calculated by the formula (14):

$$g(\mathrm{x}) = \tfrac{1}{2}(g_l(\mathrm{x}) + g_r(\mathrm{x})) \quad (14)$$

Fuzzy inference based on polynomial consequents of higher-order fuzzy rules.
The Takagi-Sugeno rule of order *n* (total *m* variables) has the following form [26]:

$$IF\ x_1\ is\ A_1^l\ AND\ x_2\ is\ A_2^l\ AND\ \ldots\ AND\ x_m\ is\ A_m^l$$

$$THEN\ y^l = a_0^l + \sum_{\substack{j_1+\ldots+j_m \le n \\ j_1, j_2, \ldots, j_m \ge 0}} \left(a^l_{x_1,x_2,\ldots,x_m}\right) x_1^{j_1}, x_2^{j_2}, \ldots, x_m^{j_m}$$

where l is the rule number in the fuzzy rule base of Takagi-Sugeno; $x_1^{j_1}, x_2^{j_2}, \ldots, x_m^{j_m}$ is a set of input variables of a fuzzy system with degrees $j_1, j_2, \ldots, j_m$; $a^l_{x_1,x_2,\ldots,x_m}$ – set of coefficients of input variables $x_1, x_2, \ldots, x_m$; a_0^l – free coefficient of rule l.

In the author's paper [26], a fuzzy inference method was proposed based on a general higher order polynomial consequent for Takagi-Sugeno fuzzy rule blocks (Fig. 2), which provides the hierarchical decomposition of an intellectual fuzzy system into separate rule blocks. However, in this case, decomposition is used only for the intermediate partition of structural elements. Only the lower tier (shown by squares) is represented in the form of blocks of fuzzy rules and, accordingly, must be calculated. Intermediate elements are abstract and used exclusively when breaking the input set into separate rule blocks. The circle in Fig. 2 denotes the conclusion of the entire fuzzy system by calculating the total consequent of order n, $x_1, x_2, \ldots, x_m$ is the set of system input values. Since the consequent is common to all blocks of fuzzy rules, the fuzzy rules themselves in the blocks at the same time consist only of the antecedents of fuzzy rules.

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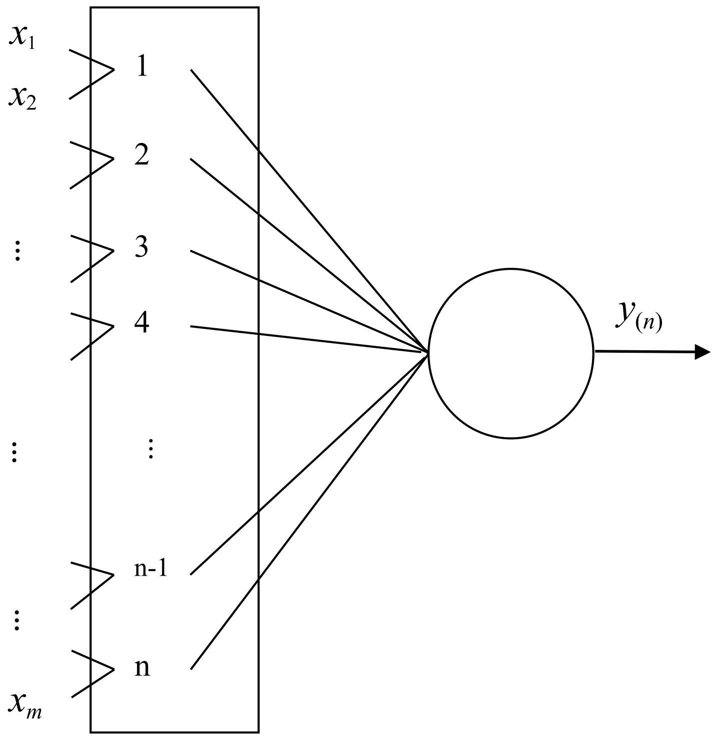

Fig. 2. The concept of the general consequent for blocks of fuzzy rules

The output function is a polynomial of order n, and is calculated only once. In other words, this function is the conclusion of the entire hierarchical system of fuzzy inference. Using the general conclusion function makes it possible to calculate the consequent separately from the antecedents of all blocks of fuzzy rules. It also opens up high opportunities for parallelism [26, 27] and leads to a decrease in the number of rules.

When using the proposed method, the output value of the fuzzy system is respectively calculated by the following formula:

$$y = \frac{y^0 \sum_{q=1}^{b}\sum_{l=1}^{p_q} \cdot \mathop{T}_{i=1}^{m_{lq}} \mu_i^{lq}(x_i) + \sum_{j=1}^{k} y^j \mathop{T}_{i=1}^{n}\left(\sum_{l=1}^{z}\mu_l(x^i_{y^j})\right)}{\sum_{q=1}^{b}\sum_{l=1}^{p_q}\cdot \mathop{T}_{i=1}^{m_{lq}}\mu_i^{lq}(x_i)+\sum_{j=1}^{k}\cdot \mathop{T}_{i=1}^{n}\left(\sum_{l=1}^{z}\mu_l(x^i_{y^j})\right)} \qquad (15)$$

where k – is the number of monomials calculated in the general consequent of the hierarchical system, b – is the total number of rule blocks, p_q – is the number of fuzzy rules of the block q, y^0 – is a free coefficient, y^j – taken separately by the function y, z – number of rules where the variable is involved $x^i_{y^j}$. As can see in (15), on the free coefficient of the system y^0 will be influenced by the whole set of rules of each block of the system. Since, without arguments, a coefficient y^0 will take place for each rule, regardless of the set of its input values in the antecedent.

A consequent of order n, consisting of monomials of all possible combinations of input variables (without repetitions), can be represented by the following formula:

$$y_{(n)}(x_1, x_2,..., x_m) =$$
$$= y_{(0)} + \sum_{i_1=1}^{m} x_{i_1}\left(y_{(1)}(x_{i_1}) + \sum_{i_2=i_1}^{m} x_{i_2}\left(y_{(2)}(x_{i_1}, x_{i_2}) + ... + \sum_{i_n=i_{n-1}}^{m} x_{i_n}\left(y_{(n)}(x_{i_1}, x_{i_2},..., x_{i_n})\right)\right)\right),$$

where $i_1, i_2,..., i_n$ are the indices of input variables by which the transition to the corresponding output of order n ($y_{(m)}$); m – number of input variables.

Reducing the order of rules in fuzzy Takagi-Sugeno systems

Since in the general case, any polynomial P from variables $x_1, x_2,..., x_m$ of order n can be represented as:

$$P(x_1, x_2,..., x_m) = k + x_1P_1 + x_2P_2 + ... + x_mP_m, \quad (16)$$

where $P_1, P_2,..., P_m$ are the polynomials of degree no greater than $n-1$. Then the consequent of rules of degree n can be represented as a combination of the corresponding consequents of degree $n-1$:

$$IF\ x_1\ is\ A_1^l\ AND\ x_2\ is\ A_2^l\ AND\ ...\ AND\ x_m\ is\ A_m^l$$
$$THEN\ y^l = k^l + x_1P_1^l + x_2P_2^l + ... + x_mP_m^l.$$

Theorem

The initial value of the fuzzy Takagi-Sugeno system of order *n* can be represented using the output values of systems of order $n-1$ as follows [26]:

$$y_{(n)} = y_{(0)} + x_1y_{(n-1)}^1 + x_2y_{(n-1)}^2 + ... + x_my_{(n-1)}^m, \quad (17)$$

where n is a fuzzy inference system order;

m – number of system variables;

$y_{(0)}$ – Takagi-Sugeno system output of order 0;

$y_{(n-1)}^i, i = \overline{1,m}$ are the system outputs of order $n-1$.

Proof of the theorem

The Takagi-Sugeno rule of order 0 can be represented as $y_0^l = a_0^l + a_1^l + a_2^l + ... + a_m^l$, that is, as the sum of the coefficients for each variable *x*. For instance:

$$y_0^l = a_0^l = a_0^l + 1_1^l + 1_2^l + ... + 1_m^l - m.$$

The general output of Takagi-Sugeno fuzzy logic system of arbitrary order can be represented as a combination of outputs, which are determined by formula (3) for each variable x_i of a fuzzy system. Then we get the following expression:

$$y = y_0 + y_2 + y_2 + ... + y_m,$$

where m is the number of variables of the fuzzy system;

$y_{(0)}$ is the output of the Takagi-Sugeno system of order 0 (taking into account only free coefficients k according to formula (16));

$y_1, y_2,..., y_m$ are the initial values for each input variable $x_1, x_2,..., x_m$.

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The overall output in this way can be represented as follows:

$$y(x_1, x_2, ..., x_m) = \sum_{i=0}^{m} \frac{\sum_{l=1}^{z} w^l y^l(x_i)}{\sum_{l=1}^{z} w^l}$$

Based on (15), we obtain:

$$y_{(0)} = y_{(0)} + y^1_{(0)} + y^2_{(0)} + ... + y^m_{(0)},$$

moreover, the initial value of order 0 is calculated as the sum of the outputs for each of the coefficients of the input values.

Suppose that for arbitrary values of x to the zero degree the following relations hold:

$$y^1 = 1(y^1) = x_1^0(y^1);$$
$$y^2 = 1(y^2) = x_2^0(y^2);$$
$$..........\ ....$$
$$y^m = 1(y^m) = x_m^0(y^m),$$

then we get:

$$y_{(0)} = y_{(0)} + x_1^0(y^1_{(0)}) + x_2^0(y^2_{(0)}) + ... + x_m^0(y^m_{(0)}),$$

that is, we represent a system of rules of order zero through the output of a zero order system. When using input variables in the first degree, the order of the block of fuzzy rules increases to the first one. Then we get an expression, the right side of which differs from the right side of the previous one only in input variables, with the degree increased by one:

$$y_{(1)} = y_{(0)} + x_1^1(y^1_{(0)}) + x_2^1(y^2_{(0)}) + ... + x_m^1(y^m_{(0)}),$$

where $y^1, y^2, ..., y^m$ – output values of order zero, that is, the order of which is one less than the order of the total initial value.

We carry out the following index transformations: changing the zero-order index by the equivalent index 1-1, we obtain:

$$y_{(1)} = y_{(0)} + x_1^1(y^1_{(1-1)}) + x_2^1(y^2_{(1-1)}) + ... + x_m^1(y^m_{(1-1)}).$$

If n is a higher order index (in this case, order 1 is higher), we obtain the following expression:

$$y_{(n)} = y_{(0)} + x_1 y^1_{(n-1)} + x_2 y^2_{(n-1)} + ... + x_m y^m_{(n-1)}.$$

Thus, the output value of a system of rules of order n is expressed in terms of the value of systems of rules of order n – 1 in accordance with (17).

The theorem is proved

Thus, representing the consequents of fuzzy rules through the consequent of order n – 1 (Fig. 3), the definition of the initial value of the Takagi-Sugeno rule system of order n is expressed by the following formula:

$$y_n = y_{(0)} + \sum_{i=0}^{m} x_i \frac{\sum_{l=1}^{z} w^l y_{(n-1)}^l (x_i)}{\sum_{l=1}^{z} w^l}$$

where m – variable number, n – rule system order, $y(x_i)$ – output value from one variable x_i, l – rule number, z – total number of rules.

It should be noted that the set of system outputs of order n – 1 is calculated independently, which means the possibility of their parallel calculation (see Fig. 3).

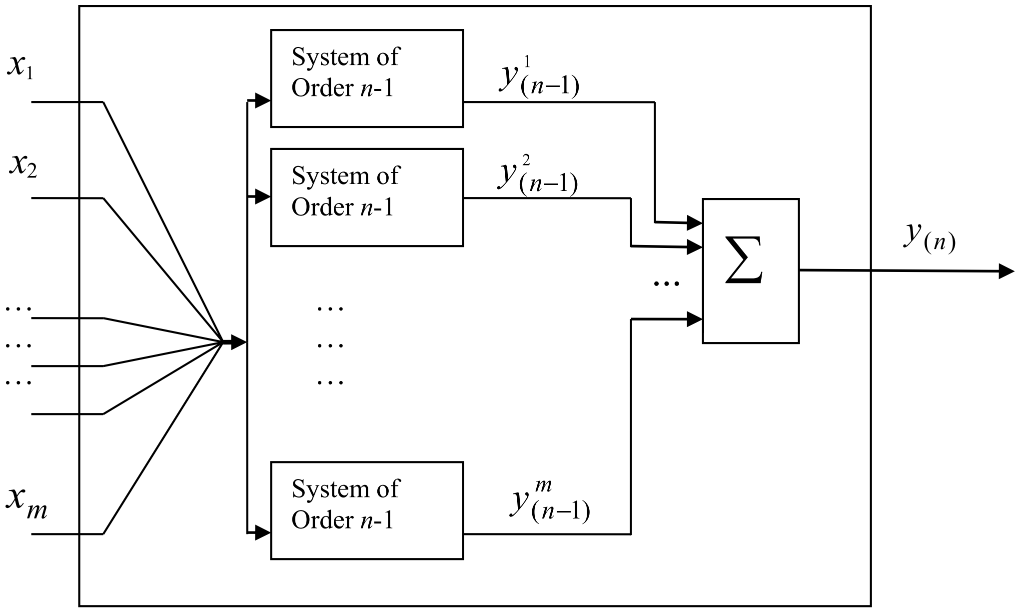

Fig. 3. Scheme of reducing the order n of a fuzzy system to order n – 1

In the case of lowering the order to the first one, we can imagine the Takagi-Sugeno fuzzy inference system in the following form:

$$y_{(n)} = \sum_{i=0}^{m} x_p \overbrace{\left(x_p \left(\ldots x_p \overbrace{\left(\frac{\sum_{l=1}^{z} w^l y_{(1)}^l}{\sum_{l=1}^{z} w^l} \right)}^{y_{(1)}} \right) \right)}^{y_{(n-1)}}$$

where y^l is the rule consequent output l; $y_{(n)}$ – n-th order fuzzy system output; $y_{(1)}$ – first order fuzzy system output; x_p – monomial consisting of the product of a subset of input variables x; l – rule number; z – total number of rules; m is the total number of variables.

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Conclusions

In the research we looked in various types of fuzzy inference systems operating on the basis of fuzzy rules with polynomial consequents, in particular Takagi-Sugeno systems. The rule antecedents of which are built on the basis of fuzzy sets of the second type. A method of higher-order fuzzy inference based on a general *n*-th-order polynomial consequent is developed, and a procedure for lowering the order of Takagi-Sugeno fuzzy rules is proposed. Which improves the accuracy of the values generated by the fuzzy inference system with fewer rules.

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