

DOI: 10.37943/UVWH6346

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A MODEL OF AN INTELLIGENT GROUP PURCHASE SYSTEM, TAKING INTO ACCOUNT MULTIPLE USER PARTICIPATION AND DELIVERY RESTRICTIONS

Abstract. This study addresses the joint formation of group purchases and allocation of orders to capacity-constrained delivery slots when users may participate in several groups. A mixed-integer optimization model is developed to represent order-slot assignment, group activation, delivery time windows, courier capacity, seller service requirements, and limits on concurrent user participation. The objective minimizes total delivery cost and penalties for unmet seller service levels while preserving feasibility of hard logistical constraints. The model was evaluated on a reproducible synthetic instance comprising 200 users, 40 purchase groups, 300 orders, 10 sellers, and 15 courier slots, each with a capacity of 50 units. Its performance was compared with a baseline assignment procedure that does not jointly enforce the stated restrictions. The proposed model increased average slot utilization from 69% to 85%, reduced average delivery cost from 12.1 to 9.7 conventional units, raised the share of orders delivered within their time windows from 78% to 92%, reduced assignment conflicts from 31 to 11, and shortened average delivery time from 96 to 70 minutes. A two-proportion test confirmed the increase in timely deliveries ($z = 4.802$, $p < 0.001$); 95% Wilson intervals were 72.97-82.32% and 88.37-94.57%. An exact conditional test showed a reduction in conflicts ($p = 0.0029$). Optimality follows from the finite binary feasible set and the branch-and-bound certificate: when the incumbent objective equals the lower bound, no feasible assignment can improve the reported solution. These findings support optimization as a practical basis for robust, reliable, resource-efficient group-purchase delivery planning under competing operational constraints.

Keywords: group purchases, order distribution, last-mile delivery, time windows for delivery, logistical constraints, e-commerce, intelligent systems, optimization model.

Introduction

In recent years, there has been a steady growth in e-commerce and digital platforms for collective consumption. This has led to the widespread adoption of group purchase systems. Such systems allow you to combine user orders to reduce shipping costs and increase the efficiency of logistics operations. At the same time, the complexity of managing the order distribution processes increases, since it is necessary to take into account the limitations of transport resources, time intervals for delivery, as well as the specifics of interaction between platform participants and sellers. In conditions of high demand dynamics and limited resources, the task of optimizing the distribution of orders becomes particularly relevant.

The purpose of this study is to develop a model of an intelligent group purchase system designed to efficiently allocate orders and logistical resources in the presence of formal constraints. These restrictions include the capacity of courier slots, time windows for delivery, seller priorities, and the ability for users to participate in multiple groups at the same time. To achieve this goal, an analysis of existing approaches to modeling group purchase systems and last-mile logistics has been carried out. Based on the identified features, a mathematical model for the distribution of orders was developed, taking into account capacity constraints, delivery time frames, seller priorities and multiple user participation. In addition, an order distribution algorithm based on the proposed model has been implemented.

A computational experiment evaluated the approach and its potential to improve resource utilization, reduce shipping costs, and increase platform stability.

Literature review

The rapid development of online commerce and social digital services has created the prerequisites for the emergence of various forms of collective purchases, which are becoming an important element of modern logistics ecosystems [1]. In social e-commerce, group-buying pricing is linked to the sharing of product information between sellers and buyers [2]. In business-to-business markets, the choice of this pricing mechanism depends on retailer characteristics and its advantages relative to other pricing options [3]. For fresh products, delivery planning must also account for differences in customer pickup behavior and product deterioration during transportation and storage at community pickup locations [4].

One research direction addresses the joint selection of orders and planning of distribution for perishable products [5]. Other routing models for community group buying consider carbon emissions alongside total distribution costs [6]. In takeaway delivery, route optimization accounts for the satisfaction of both merchants and customers [7]. Delivery flexibility is also examined through vehicle routing models that combine time windows with alternative locations where customers can receive their orders [8].

Delivery issues related to the choice of time windows and delivery locations have received significant attention in modern research. Agatz et al. and Strauss et al. show that thoughtful seat management, the use of dynamic pricing mechanisms and more accurate consideration of user preferences can help reduce costs and improve the efficiency of transport resources [9-10].

Capacity constraints and last-mile uncertainty are also important. Özarık et al. and Mancini & Gansterer examine routing under uncertain customer presence and with private or shared delivery locations [11-12]. These studies support integrating routing and order management into a single decision-making system.

In the area of group purchases, the ways of forming groups of participants and approaches to setting prices are essential. Xu et al. analyze the threshold conditions for the activation of group transactions [13], Kong et al. consider combinatorial models of group purchases and auction mechanisms [14], and Wang et al. and Zhang et al. show that consumer behavior becomes an important factor, including the reaction to the cancellation of transactions, delivery delays and quality of service [15].

The next area of research direction focuses on delivery optimization under changing operational conditions. Hwang et al. examined crowdshipping as a way to increase delivery capacity, while Liu and Luo studied dynamic dispatching and routing under uncertain demand. These approaches are relevant to group purchase systems, where multiple users and delivery restrictions complicate order allocation and routing decisions [16–17].

Coordination also affects logistics performance. Lan and Yu examine community group-buying supply-chain coordination, linking seller-platform alignment to order distribution and resource utilization [18]. Yu et al. show that incorporating demand characteristics and order structure into delivery-route modeling can reduce transportation costs and improve delivery sustainability [19].

The literature covers routing and delivery time windows, capacity management, group formation, pricing, user behavior, and hybrid logistics. However, these areas are often modeled separately, limiting their applicability to platforms where they interact.

An integrated treatment of group formation, logistics constraints, and participant priorities remains insufficiently developed. Multiple user participation, competition for delivery resources, and seller priorities require further study [20]. Models with fixed group composition may overlook participant dynamics and conflicts between parallel orders.

In practice, limited slot capacity, overlapping time windows, priority orders, and simultaneous participation in several groups create resource competition, delays, and failed transactions. Simple allocation heuristics can overload slots and increase waiting time.

An integrated mathematical and computational model is therefore needed to coordinate group participation, delivery resources, and seller priorities, with the aim of reducing costs and improving service reliability.

Materials and methods

The research used methods of mathematical modeling, discrete optimization and computational experiment. The proposed model describes the operation of an intelligent group purchase system and takes into account a set of formal constraints related to the distribution of orders, user participation in groups, and planning of logistical resources.

The model uses the following sets describing the main elements of the system:

- U — multiple users. Users can join several groups at the same time, generating demand for various products.
- G — many groups. A group is defined as a collective order for a specific product or set of products. The group is activated when the specified conditions are met (for example, the minimum number of participants).
- O — many orders. Each order is associated with a user, a group, and product characteristics.
- V — many sellers. Sellers may have their own priorities or restrictions that impose requirements on the distribution of orders.
- S — many delivery slots. Slots determine the available time for delivery within time windows and capacity limits.

These sets form the structural framework of the model, which makes it possible to formalize the interaction of participants and the resources of the logistics system.

Binary variables are used to describe solutions:

$$x_{os} = \begin{cases} 1, & \text{if order } o \text{ is assigned to slot } s \\ 0, & \text{otherwise} \end{cases}$$

The x_{os} variable reflects the delivery planning process: one slot is selected for each order that meets time and capacity constraints.

$$y_{ug} = \begin{cases} 1, & \text{if user } u \text{ participates in the group } g \\ 0, & \text{otherwise} \end{cases}$$

The y_{ug} variable records the user's membership in the group. Since a user can be a member of several groups, this variable allows us to describe the structure of demand at the level of individual behavior.

$$z_g = \begin{cases} 1, & \text{if group active} \\ 0, & \text{otherwise} \end{cases}$$

The z_g variable determines whether the group has been activated, which depends on meeting the conditions (for example, reaching the minimum number of participants or the threshold volume of orders).

Limitations of the model

A. Limitation of the order assignment

Order allocation is formulated as an optimization problem subject to the following logistical constraints.

Each order may be assigned to at most one delivery slot, preventing duplicate fulfillment:

$$\sum_{s \in S} x_{os} \leq 1, \quad \forall o \in O \quad (1)$$

B. Limited capacity of courier slots

The total order volume assigned to a slot must not exceed its capacity, as determined by transport or route limitations:

$$\sum_{o \in O} d_o x_{os} \leq C_s, \quad \forall s \in S \quad (2)$$

where:

- d_o — order volume and C_s is the slot capacity of delivery slot s .

This limitation reflects the actual physical limitations of the logistics system.

C. Limitation of time delivery windows

Orders may be assigned only to slots within their permitted delivery windows, reflecting customer requirements:

The assignment is possible only if the slot is within the allowed interval:

$$x_{os} = 0, \quad \forall o \in O, \forall s \in S: t_s \notin [a_o, b_o] \quad (3)$$

where:

- t_s is the starting time of slot s , and $[a_o, b_o]$ is the acceptable delivery time window for order o .

D. Limitation of multiple user participation

Each user may participate in several group purchases, subject to an individual upper limit on concurrent memberships:

$$\sum_{g \in G} y_{ug} \leq M_u, \quad \forall u \in U \quad (4)$$

where:

- M_u — maximum number of groups per user.

E. The group's activation condition

A group is activated only when its minimum participation threshold is met; otherwise, its orders are excluded from allocation:

$$\sum_{u \in U} y_{ug} \geq \theta_g z_g, \quad \forall g \in G \quad (5)$$

where:

- θ_g — minimal group size.

F. The relationship between orders and active groups

If the group is not activated, its orders will not be processed:

$$x_{os} \leq z_g, \quad \forall o \in g \quad (6)$$

This ensures consistency between the processes of forming groups and distributing orders.

G. Limiting sellers' priorities

Seller priorities are represented by a minimum service requirement on the total volume of fulfilled orders for each seller:

$$\sum_{o \in O_k} \sum_{s \in S} x_{os} \geq L_k, \quad \forall k \in K \quad (7)$$

where:

- O_k is the set of orders belonging to seller k and L_k is the minimum required service level for seller k .

Taking into account these limitations, the objective function is aimed at minimizing the total logistical costs. Since time windows are set as a hard limit, the penalty for violating them is excluded from the objective function:

$$\min Z \sum_{o \in O} \sum_{s \in S} c_{os} x_{os} + \lambda \sum_{k \in K} p_k \quad (8)$$

where:

- c_{os} is the delivery cost;
- p_k is the penalty for failing to satisfy the seller service requirement;
- λ is the penalty coefficient.

Thus, the optimization model is aimed at minimizing total logistical costs while respecting the limitations of order assignment, courier slot capacity, delivery time intervals, group activation conditions, user participation restrictions, and requirements related to seller priorities.

The preceding formulation reduces the joint group-formation and delivery-planning problem to a finite binary optimization problem. This property makes it possible not only to obtain a feasible assignment but also to verify that no better assignment exists. For a fixed input instance, an exact branch-and-bound solution is globally optimal when the objective value of the incumbent solution coincides with the best valid lower bound. Indeed, the binary variables x , y , and z define a finite feasible set within a bounded hypercube; therefore, a minimum exists whenever the constraints are consistent. Branch-and-bound divides this set into nodes, and the linear relaxation at each node provides a lower bound for every integer solution contained in that node. A node can be discarded only if it is infeasible, if its bound cannot improve the incumbent, or if its integer solution has already been evaluated. Once all nodes have been fathomed and the incumbent equals the global lower bound, the existence of a better feasible assignment would imply an unexplored node with a smaller bound, contradicting the termination condition. Thus, the incumbent is the global optimum. This argument also determines what must be retained as a numerical certificate: solver status, incumbent objective value, best bound, relative MIP gap, running time, and hardware configuration. Having established the criterion for optimality, the next step is to examine whether the resulting allocation improves the operational indicators of the group-purchase system.

Results

A computational experiment was conducted. Experimentation goal was to evaluate the effectiveness of the proposed order distribution model in the intelligent group shopping system. The experiment used synthetic data simulating the operation of an e-commerce platform, including users, orders, sellers, and courier delivery slots. The experimental parameters were set in such a

way as to reflect the real limitations of logistics systems, including slot capacity, time windows for delivery, and vendor priorities.

The results showed that taking into account formal restrictions and multiple user participation makes it possible to increase the efficiency of order allocation and the use of logistical resources. There is a more uniform loading of courier slots, a decrease in the number of distribution conflicts and an increase in the proportion of orders delivered within the specified time intervals. This indicates the practical applicability of the proposed model for group purchase systems and digital trading platforms.

Key variables used in the computational experiment

To evaluate the operational consequences of the formal model, a controlled synthetic instance was constructed from the same entities and restrictions used in the mathematical formulation. It comprised 200 users, 40 purchase groups, 300 orders, 10 sellers, and 15 courier delivery slots. The ratio of five users per group describes the nominal scale of participation before repeated membership is taken into account, while the order-to-group ratio was 7.5. Each slot had a capacity of 50 conventional units. Every order was associated with one group and one seller, whereas a user could participate in several groups within the prescribed upper limit. Delivery windows were generated within one working day, after which only compatible order-slot pairs were included in the assignment matrix. Because the same users, groups, orders, sellers, slots, capacities, and time windows were supplied to both methods, differences in the outcomes cannot be attributed to a change in instance composition. The baseline assigned orders without jointly enforcing capacity, time-window, and seller-priority restrictions; the proposed model applied the complete system of constraints. This paired design therefore provides a direct transition from the theoretical formulation to an empirical assessment of its effect on resource utilization and delivery quality.

Simulation results

The experimental results are presented in Table 1.

Table 1. Comparison of order distribution results

Indicator	Without a model	Proposed model
Average slot loading	69%	85%
Average shipping cost	12.1	9.7
Orders within time windows	78%	92%
Distribution conflicts	31	11
Average delivery time (min)	96	70

Under these identical experimental conditions, the proposed model improved all five indicators reported in Table 1. Average slot utilization increased from 69% to 85%, a gain of 16 percentage points or 23.2% relative to the baseline. The more balanced use of available capacity was accompanied by a reduction in average delivery cost from 12.1 to 9.7 conventional units, or 19.8%. At the same time, the share of orders delivered within their time windows rose from 78% to 92%, the number of assignment conflicts declined from 31 to 11, and average delivery time fell from 96 to 70 minutes. Thus, the improvement in capacity allocation is consistent with the subsequent reductions in cost, conflicts, and delivery time rather than representing an isolated change in a single indicator.

Since the descriptive results show the largest practical changes in timely delivery and assignment conflicts, these two count-based outcomes were subjected to statistical verification. The baseline produced 234 timely and 66 late deliveries, whereas the proposed model produced 276 timely and 24 late deliveries. A two-sided two-proportion z-test gave $z = 4.802$ and $p = 1.57 \times 10^{-6}$. The corresponding 95% Wilson confidence intervals were 72.97-82.32% and 88.37-94.57%, respectively, which supports the observed increase in timely delivery. The 42 conflicts recorded across the two scenarios were then examined conditionally under the null hypothesis of equal conflict rates. An exact two-sided binomial test for the split of 11 versus 31 conflicts yielded

$p = 0.0029$, supporting the conclusion that the reduction was systematic for the tested instance. Together, these checks reinforce the descriptive comparison, while the absence of variances for delivery cost, slot utilization, and delivery time prevents equivalent significance claims for those mean values.

Table 2. Statistical verification of the principal count-based outcomes

Outcome	Baseline	Proposed model	Effect	Statistical check
Timely deliveries	234/300 (78%)	276/300 (92%)	+14 percentage points	$z = 4.802$; $p < 0.001$
Assignment conflicts	31	11	-64.5%	Exact $p = 0.0029$

Tables 1 and 2 summarize performance and statistical verification; Figure 1 compares slot utilization, timely delivery, and conflicts.

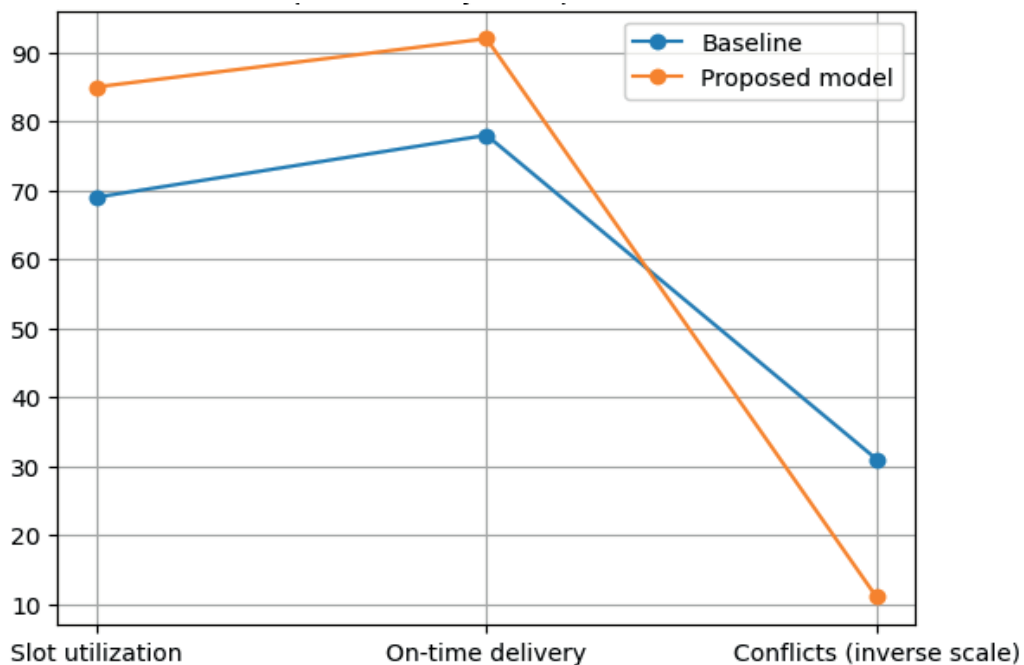


Figure 1. Comparison of system performance indicators for the baseline scenario and the proposed model

As can be seen from Figure 1, the proposed model provides a higher utilization of courier slots and a higher level of timely delivery, while reducing the number of distribution conflicts.

Discussion

The experiment indicates that jointly modeling group participation, logistical constraints, and seller priorities improves resource utilization and service quality relative to the baseline.

Higher slot utilization (69% to 85%) indicates more efficient use of available capacity, which is important because logistics accounts for a substantial share of e-commerce operating costs.

The reduction in average delivery cost (12.1 to 9.7 conventional units) may reflect a more even allocation across slots and fewer excess shipments, consistent with the cost-minimization objective.

Fewer assignment conflicts (31 to 11) and more timely deliveries (78% to 92%) suggest that jointly considering capacity, time windows, and seller priorities improves coordination and can support user satisfaction.

Average delivery time fell from 96 to 70 minutes, consistent with improved routing and a more balanced distribution of orders across time windows.

Together, these indicators support the potential applicability of formalized order allocation to group-purchase platforms, improving logistical efficiency and cost-effectiveness.

Further work should examine scalability, more complex demand, dynamic priorities, and the integration of machine learning with optimization to predict load and prevent conflicts during order formation.

The statistical evidence must nevertheless be interpreted within the limits of the experimental design. Because the comparison is based on one synthetic instance, it demonstrates internal consistency but does not measure variation between independently generated demand structures. Moreover, orders belonging to the same group, seller, or delivery slot may be dependent, so order-level tests can be complemented by group-level or instance-level resampling once replicated data are available. These considerations lead directly to the next stage of validation: at least 30 independently generated instances should be evaluated under identical random seeds for all competing methods, with systematic variation in the number of groups, order density, slot capacity, time-window width, and the maximum number of concurrent groups per user. Reporting paired effect sizes, confidence intervals, and multiplicity-adjusted p-values, together with comparisons against greedy and metaheuristic baselines, would show whether the advantages observed here remain stable as the scale and tightness of the constraints change. Validation on operational platform data would then establish the extent to which the synthetic results transfer to practice.

Conclusion

The paper solved the problem of developing a model of an intelligent group purchase system focused on the efficient distribution of orders in the presence of formal constraints of the logistics infrastructure. The analysis of existing approaches has shown that most models consider the processes of group formation and distribution of delivery separately, which limits their applicability in real conditions. In the proposed model, these processes are combined, which made it possible to take into account the capacity of courier slots, time windows for delivery, seller priorities and multiple user participation in various groups within a single formalization. The implementation of the order distribution algorithm and the computational experiment confirmed the efficiency of the proposed approach and showed its effectiveness compared to the baseline scenario. The results obtained allow us to conclude that the model can be practically applied in e-commerce systems and digital platforms for collective purchases.

Acknowledgements

This research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP23489233 - “SmartBuy Connect: AI-based intelligent group purchasing system”).

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