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## A MATHEMATICAL MODEL FOR AN AUTOMATED SYSTEM OF MEDICAL DIAGNOSTICS

**Abstract:** One of the primary focuses of the Republic of Kazakhstan concerning sustainable and stable improvements in the well-being of its population is the advancement of the healthcare sector. A mathematical model for an automated medical diagnostics system integrates machine learning algorithms, statistical models, and decision trees to analyze patient data and facilitate accurate diagnoses. This model enables healthcare professionals to enhance the efficiency and reliability of medical diagnostics by leveraging advanced computational techniques. These distinguishing features can be incorporated by developing a mathematical model for diagnosing diseases, enabling precise identification, and guiding appropriate treatment strategies.

Machine learning algorithms play a crucial role in automated systems for medical diagnostics. An ensemble of multiple algorithms, such as combining decision trees with gradient boosting or using a combination of neural networks and traditional machine learning, can yield improved diagnostic accuracy and robustness. Predicting the progression of diseases is a crucial aspect of healthcare, enabling personalized interventions and improved patient outcomes. A mathematical approach can facilitate this prediction by monitoring changes in diagnostic results aligned with the severity of symptoms, which inherently vary over the observation period. By employing mathematical modeling techniques, healthcare professionals gain valuable insights into disease progression, supporting informed decision-making and tailored treatments.

In conclusion, developing a mathematical model for an automated medical diagnostics system, incorporating machine learning algorithms, statistical models, and decision trees, significantly contributes to healthcare. These models enhance the accuracy, efficiency, and personalization of medical diagnoses. Additionally, mathematical models aid in the differential diagnosis of challenging conditions and provide predictions regarding disease progression, ultimately benefiting patient care and treatment outcomes.

**Keywords:** IT in medicine, diagnostic model, mathematical decision-making method, modeling

## Introduction

In recent years, mathematical methods have played an increasingly important role in medical research and Diagnosis. The use of mathematical methods in medical research and Diagnosis can revolutionize the field and improve patient outcomes by enabling more accurate diagnoses, faster treatments, and more personalized care [1, 2].

The research paper describes the development of a mathematical model of Diagnosis based on the gradient projection method. This approach can reduce subjectivity and increase objectivity in medical decision-making, essential for accurate Diagnosis and effective treatment. The use of fuzzy logic in the initial information of the model is also promising, as it allows for the incorporation of uncertain or imprecise data that may be encountered in medical Diagnosis. This can help improve the model's accuracy and reduce the risk of misdiagnosis [3, 4].

In addition, using the model as a teaching element in the educational process can benefit students and medical professionals. It can improve their understanding of the diagnostic process and enable them to make more informed and accurate decisions when diagnosing patients [5].

## Related Works

In the realm of medical research, the development of mathematical models for diagnosis holds immense promise, offering the potential to significantly enhance patient outcomes and propel advancements in the field of medicine. These mathematical models, designed for automated medical diagnostic systems, encompass a range of techniques, including machine learning algorithms, statistical models, and decision trees [6]. Their primary objective is to scrutinize patient data comprehensively, taking into account vital factors such as medical history, symptoms, age, and other pertinent information, to arrive at accurate diagnoses.

The integration of a mathematical approach into medical research opens up novel avenues for tackling diagnostic challenges. In the diagnostic process for various medical conditions, a sequential and logically driven approach is followed, often employing diagnostic algorithms that leverage mathematical models tailored to medical logic. Notably, the utilization of fuzzy set theory techniques exemplifies the potential of this approach in creating information-mathematical models for diagnosis and prediction.

A model is a concise representation of either real or virtual objects or systems that enables the study of their essential properties, parameters, and characteristics. In the field of modeling theory, three types of models are considered [7]:

- Experimental models: These models describe systems by establishing a relationship between inputs and outputs using the “black box” method. The focus is on obtaining dependencies between the information and corresponding output variables through experimental observations.
- Experimental-analytical models: These models partially reflect the physical nature of the original system by decomposing complex phenomena into simpler components. The model of the entire system is adjusted to a known law by incorporating effective coefficients obtained from experimental data. This type of model combines experimental observations with analytical techniques to provide a more detailed representation of the original system.
- Theoretical models: These models are constructed based on mass, energy, and momentum conservation principles. They aim to reflect the physical nature of phenomena and processes comprehensively. Theoretical models are developed by incorporating detailed knowledge and understanding of the system's underlying physical regulations.

Each model type serves a specific purpose and offers different levels of detail and accuracy [8, 9]. Experimental models focus on empirical observations, while experimental-analytical models provide a simplified representation with adjusted coefficients. Theoretical models, on the other hand, strive to capture the physical nature of the system by utilizing fundamental principles.

By employing these various models, researchers and scientists can gain insights into the original system's behavior, properties, and characteristics, facilitating analysis, prediction, and understanding of complex phenomena [10].

The primary focus of modeling is to investigate and comprehend the dynamics and responses of organizational and technical systems. By modifying the characteristics of the elements and connections within these systems, researchers and practitioners can simulate various scenarios and assess the resulting behavior and outcomes. Modeling helps gain insights into the dependencies, interdependencies, and interactions between different systems components, providing a basis for informed decision-making and effective management [11, 12].

Modeling plays a crucial role in comprehending complex organizational and technical systems. It enables a systematic analysis of these systems by employing the system approach, allowing for exploring different scenarios and evaluating their potential implications.

To manage any object, knowledge or information about the current state and possible changes in system parameters is required. In general, some knowledge of four parameters characterizes A:

$$A = \{ \text{object; property; value; time} \} \quad (1)$$

This information, the value of a given property at a certain time, can be obtained by direct measurement of a real object - the original. However, it is only sometimes possible to perform such a measurement for real objects, especially poorly structured systems. In addition, for systems that are in the design phase, it is, by definition, impossible to measure parameters (1).

Unlike direct measurements, modeling methods provide a means to acquire new knowledge about an object and explore the fundamental laws governing the behavior of objects under different conditions, system characteristics, and control influences [13].

In the context of automated control and design systems, the model serves as a substitute for the original object. Numerous definitions exist for the concept of a model, but one of them states that a model is an entity created by a researcher that accurately represents the essential properties of the original. Models can be developed for real technical systems as well as for systems that only exist in our imagination or may be created in the future. In other words, the synthesis and examination of the model can precede the creation of the original. Multiple models can be created for any given original, differing in the number of parameters considered and the approach used for their development [14]. The model should accurately reflect the essential properties of the original, which are necessary and sufficient to address the research objectives at hand.

Concept of essentiality of the model. The property of essentiality of models follows from the relationship of homomorphism and isomorphism. If only part of the properties of the original is reflected in the model, then it is said to be a homomorphism. With mutual homomorphism, when all the properties of the original are reflected in the model, and all the properties of the model are of the original, we speak of the isomorphism of the model of the original. A model that reflects such properties of the original that are necessary for solving a scientific or practical problem is essential (fig. 1).

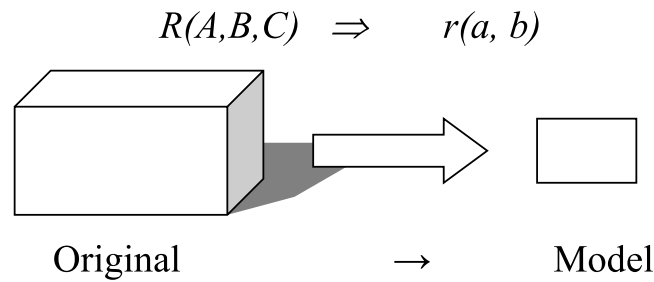


Figure 1. The properties of the original in the model

In Fig.1, the original object has three dimensions, whereas its model represents only two. This establishes a homomorphism relationship between the original and the model, indicating that the model captures some, but not all, of the properties of the original. The model is simplified compared to the original object. The key factor in determining the essentiality of a model is its ability to capture the relevant features and relationships necessary for solving a specific problem or achieving a particular objective. As long as the model adequately represents the essential aspects required for the problem's solution, it can be considered essential, despite its simplifications or deviations from the complete set of properties exhibited by the original object.

Requirements for the created models: In scientific practice, the following procedure for working with various models of objects is generally accepted [15]:

- building the model and checking its performance
- carrying out experiments using the model
- analysis of the obtained results
- development of recommendations and methods of transferring results to the original.

At the same time, the main requirements for the created models are

- the model must be substantial and should be easier to create than the original;
- the model should be cheaper than the original;
- there must be rules according to which the results obtained on the model are transferred to the original.

The isomorphism of the mathematical description allows you to transfer the results obtained when solving one equation to a whole class of objects with a similar description. The similarity of mathematical descriptions gives mathematical models the property of universality. If the mathematical description of the original corresponds to the model, then the model fully reflects the essential properties of the original.

The use of models in scientific investigation and system diagnosis is common practice in scientific research to consider both direct and inverse modeling tasks. The setting of the investigation when the starting parameters of the item are established as a consequence of modeling is a distinctive feature for assignments involving direct modeling.

The task of figuring out the object's input parameters is specified in reverse modeling tasks [16]. Research and design of organizational and technical systems optimization. The general task of modeling is to establish dependence:

$$Y = f(X, U, Z) = \varphi(X, U) + \xi(Z), \quad (2)$$

where  $(X, U)$  – object display;  $(Z)$  – model error.

The diagram (fig. 2) shows a generalized scheme of the object. In research tasks, direct tasks are solved with the help of models: «Given the given values of input parameters  $X$  and control actions  $U$ , find the value of the objective function  $Y$ ». The specified control scheme

allows you to realize the system's reaction to possible disturbances in the values of the input parameter  $X$ .

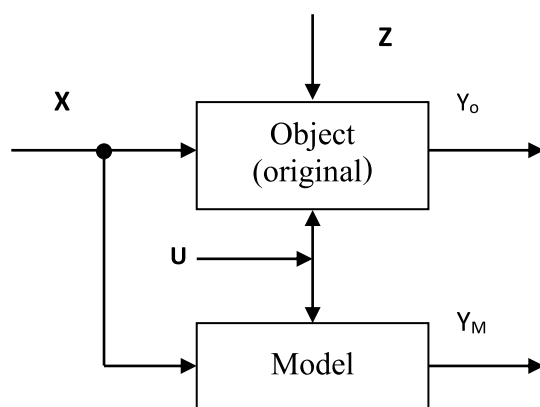


Figure 2. Scheme of control of the object by disturbance:  
 $X$  – vector of input parameters;  
 $Y_o$  – object output parameters (goal function);  
 $Y_M$  – initial parameters of the object  
 $U$  – vector of control actions;  
 $Z$  – uncontrolled disturbances.

In organizational and technical control systems, models are used, as a rule, for proactive control based on anticipatory forecasting of the system state. Models are one of the elements of the control circuit. In this case, the task is: «Find such values of input parameters  $X$  and control  $U$ , which provide the necessary value  $Y$ ».

Within the realm of healthcare and scientific literature, diagnostic tables serve as structured databases categorizing relevant diseases within specific medical classes [3, 17]. These diagnostic tables incorporate a wide array of diagnostic indicators, which can be both quantitative (such as body temperature, blood pressure, and erythrocyte sedimentation rate) and binary (where the parameter is categorized as either normal or abnormal). Additionally, these indicators can encompass vague or imprecise expressions, such as slight skin reddening or severe heart region pain. Furthermore, it is possible to quantify the degree of correlation between various symptoms and their corresponding diseases [18].

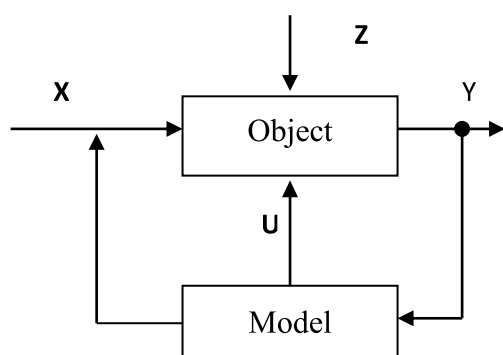


Figure 3. Object diagnosing scheme for deviations.

*Machine learning algorithms* play a pivotal and transformative role in disease diagnosis across diverse medical domains. These algorithms leverage the potency of data analysis to discern intricate patterns, make predictive assessments, and categorize diseases through the analysis of patient data, medical imagery, and relevant medical information. One prominent example of this application is the utilization of Neural Networks, especially deep learning models, in numerous medical imaging tasks. These tasks encompass the detection of diabetic retinopathy, identification of tumors in radiological images, and diagnosis of lung diseases using CT scans. These complex, multilayer networks are adept at acquiring and understanding intricate data patterns and representations [19].

In addition to Neural Networks, K-Nearest Neighbors (KNN) algorithms also contribute significantly to medical applications, particularly in disease risk prediction and patient stratifica-

tion. KNN accomplishes this by classifying patients based on the majority class among their k-nearest neighbors within the feature space [20]. This approach enhances our ability to make informed decisions regarding disease risk and appropriate patient management.

In the realm of text-based disease diagnosis, Naive Bayes algorithms find their application by leveraging electronic health records or medical reports. These algorithms make use of Bayes' theorem to calculate the probability of a particular diagnosis based on the observed features within the text [21]. This approach aids in the automated identification of diseases from textual medical data.

On the other hand, Gradient Boosting Algorithms, exemplified by XGBoost and LightGBM, are harnessed for the purpose of disease classification and the early detection of chronic conditions. These ensemble methods work by combining multiple weaker models to significantly improve predictive accuracy [22]. By aggregating the outputs of these weaker models, Gradient Boosting Algorithms excel in distinguishing and classifying various diseases, contributing to early diagnosis and effective disease management. We believe that finding the right diagnosis cannot be achieved simply by extracting symptoms and running a classification algorithm. A more accurate approach is to collect related symptoms by linking some events to other events and symptoms that are claimed by the patient or the doctor following the patient. We used a polynomial naive Bayesian classifier, which was trained on a selected data set, which is freely available under CC4.0 [23]. The polynomial naive Bayesian classifier uses a multinomial distribution to model the frequency of occurrence of words in texts. The scikit-learn library was used for implementation in Python. The pre-trained data for using the Glove model [24] were used on specific manifestations of symptoms determined using fuzzy logic methods.

*The differential diagnosis between primary biliary cirrhosis of the liver and active hepatitis with the cholestatic syndrome can be challenging due to the similarity of their symptoms. Both conditions can present with jaundice, pruritus, fatigue, and hepatomegaly. However, some clinical and laboratory features can help to distinguish between them [25]. In diagnosing diseases, the differential diagnosis between primary biliary cirrhosis of the liver and active hepatitis with cholestatic syndrome poses a challenge due to their overlapping symptoms. Both conditions exhibit jaundice, pruritus, fatigue, and hepatomegaly. However, distinguishing features based on clinical and laboratory observations can aid in accurate differentiation between the two states.*

Mathematical decision-making methods with qualitative uncertainty, such as the Bellman-Zadeh approach, can be very useful in fields such as medicine, where there is often incomplete or uncertain information. Fuzzy logic, a branch of mathematics that deals with uncertainty, can be used to model imprecise or vague information and provide a more accurate representation of the real world. By using fuzzy logic, decision-making models can incorporate subjective human judgment and expert knowledge systematically and rigorously. Several studies have demonstrated the effectiveness of fuzzy decision-making models in medical applications, such as diagnosing diseases and selecting treatments. These models can help doctors make more informed decisions and improve patient outcomes [26, 27].

However, it is important to note that these models are separate from medical expertise and clinical judgment. They should be used as a tool to support and enhance the decision-making process, not replace it entirely [28].

### **Developing a mathematical model for diagnosing diseases**

By using fuzzy logic and decision-making methods, the model can take into account the imprecise and uncertain nature of diagnostic signs and combine them with expert knowledge to make accurate and reliable diagnoses. To develop such a model, a number of steps were taken, such as:

1. Identifying the diagnostic signs and symptoms associated with enteric and pancreatic insufficiency and gathering data on their frequency, severity, and other relevant characteristics.

2. We are formulating the diagnostic problem as a decision-making problem, with the goal of making a diagnosis based on the available diagnostic signs.

3. It is developing a set of fuzzy rules that incorporate the expert knowledge and diagnostic signs to make a diagnosis.

4. We are implementing the fuzzy decision-making model using appropriate software tools and validating the model using clinical data.

5. It evaluates the performance of the model, compares it to other diagnostic methods and assesses its usefulness in clinical practice.

Overall, developing a mathematical model for diagnosing diseases of enteric and pancreatic insufficiency using fuzzy decision-making methods has the potential to improve the accuracy and reliability of diagnoses, leading to better patient outcomes and more effective treatment strategies.

The table below represents the differential diagnosis between primary biliary cirrhosis of the liver and active hepatitis. Signs of disease are described in the table, and they are objective indicators of a person's health status that can be observed or measured by a healthcare provider. These signs can include physical, biochemical, or physiological measurements, and they can be used to diagnose or monitor a disease's progress. There is considered a standard reference for internal medicine and covers a wide range of medical topics, including liver and biliary tract diseases. The chapter on liver and biliary tract diseases provides in-depth coverage of the differential diagnosis between primary biliary cirrhosis (PBC) of the liver and active hepatitis, as well as other liver diseases.

Table 1. The differential diagnosis between primary biliary cirrhosis of the liver and active hepatitis

<b>Signs of disease</b>	<b>Primary biliary cirrhosis of the liver</b>	<b>Active hepatitis with cholestatic syndrome</b>
Serum immunoglobulin content	Increased, dominated by level up IgM	Increased, dominated by level up IgM (irregularly)
Determination of high titer antimitochondrial antibodies in the blood	Very typical	Not typical
Determination of antibodies to smooth muscles in the blood	Little typical	Very characteristic of autoimmune hepatitis
High blood levels of alanine aminotransferase	Observed, but not dominant compared to biochemical cholestasis syndrome	Very characteristic and dominant in comparison with the biochemical syndrome of cholestasis
Needle biopsy of the liver	The predominance of lesions of the bile ducts over changes in the liver parenchyma, destruction of the interlobular and septal ducts, and the development of peripheral cholestasis are characteristic	The predominance of lesions of the parenchyma and the presence of necrosis in the hepatic lobules are characteristic; the phenomena of cholestasis and destruction of the bile ducts are less pronounced
Portal hypertension syndrome	Characteristic in the advanced stage of the disease	Not typical

Table 2. Utility matrix for specified diseases

Primary biliary cirrhosis of the liver	X	OX	KP	HX	X	X
Active hepatitis with cholestatic syndrome	KP	HX	OX	OX	X	HX

Patient's symptoms:

- 1 the content of immunoglobulin in the blood serum is increased;
- 2- detected in the blood antimitochondrial antibodies in high titer.
- 4- there is a high level of alanine aminotransferase in the blood, but it does not dominate in comparison with the biochemical syndrome of cholestasis.
- 6- there is a portal hypertension syndrome in the advanced stage of the disease.

Table 3. Utility matrix for specified symptoms:

A (desiases)	X1	X2	X4	X6
A1	X	OX	KP	X
A2	KP	HX	OX	HX

where A1 – Primary biliary cirrhosis of the liver; A2 – Active hepatitis with cholestatic syndrome, and utilities are set qualitatively: OX – very typical; HX – uncharacteristically; X – characteristically; KP – rarely. Utilities given in the form of linguistic variables can be represented as subsets:

$$OX = \{0.5/9; 1.0/10\}; X = \{0.5/7; 1.0/8\}; HX = \{0.5/2; 1.0/3; 0.5/4\}; KP = \{0.5/2; 1.0/1\}.$$

The expert sets the index, which accounts for the maximum degree of membership. The following set represents the patient's state:

$$X = \bigcup_k \mu \sim (X_k) / X_k \quad (3)$$

where  $x_k \in X$  and  $\mu \sim (x_k)$  – the severity of that symptom.

$$\{0.5/X1; 0.7/X2; 0.5/X4; 0.9/X6\}$$

To solve the problem, we form a matrix:

$$U = \begin{bmatrix} u_{11} & \dots & u_{1n} \\ u_{21} & \dots & u_{2n} \\ \dots & & \dots \\ u_{m1} & \dots & u_{mn} \end{bmatrix}$$

where  $\tilde{U}_{ij} = \bigcup_k \mu_{u_{ij}}(u_k) / u_k$ , and  $u_k = \mu \sim (x_k)$ , are a fuzzy set defined by:

$$\mu_{u_i}(\tilde{u}_k) / u_k = [\mu(u_i) / u_i] / [\mu_{u_k}(u_i) / u_i] \quad (4)$$

$$\{0.5/X1; 0.7/X2; 0.5/X4; 0.9/X6\}$$

$$U_1 = \{0.5/X; 0.7/OX; 0.5/KP; 0.9/X\}$$

$$U_2 = \{0.5/KP; 0.7/HX; 0.5/OX; 0.9/HX\}$$

Let's transform it and combine the same symptoms,

as  $(0.5/X + 0.9/X - (0.5/X * 0.9/X) = 0.95/X)$  and  $(0.7/HX + 0.9/HX - (0.7/HX * 0.9/HX) = 0.97/HX)$



then:

$$U_1 = \{0.95/X; 0.7/OX; 0.5/KP;\}$$

$$U_2 = \{0.5/KP; 0.97/HX; 0.5/OX;\}$$

$$U_1 = \{0.95/(0.5/7, 1.0/8); 0.7/(0.5/9, 1.0/10); 0.5/(0.5/2, 1.0/1);\}$$

$$U_2 = \{0.5/(0.5/2, 1.0/1); 0.97/(0.5/2; 1.0/3; 0.5/4); 0.5/(0.5/9, 1.0/10);\}$$

Then  $\tilde{u}_i$  simplify as follows:  $\mu_{\tilde{u}_i}(u_k) = \min[\mu_x(x_k), \mu_{u_{ik}}(u_i)]$

Minimization leads to a reduced risk of misdiagnosis.

$$U_1 = \{0.5/7, 0.95/8, 0.5/9, 0.7/10, 0.5/2, 0.5/1\}$$

$$U_2 = \{0.5/2, 0.5/1, 0.5/2, 0.97/3, 0.5/4, 0.5/9, 0.5/10\}$$

Find:

$$u_{\max} = \sup Y, \text{ где } Y = S(U_1) \cup S(U_2) = \{1,2,3,4,7,8,9,10\}; \quad u_{\max} = 10;$$

Maximizing set:

$$U_{1m} = \{7:10/7, 8:10/8, 9:10/9, 10:10/10, 2:10/2, 1:10/1\}$$

$$U_{2m} = \{2:10/2, 1:10/1, 2:10/2, 3:10/3, 4:10/4, 9:10/9, 10:10/10\}$$

$$U_{1m} = \{0.7/7, 0.8/8, 0.9/9, 1/10, 0.2/2, 0.1/1\}$$

$$U_{2m} = \{0.2/2, 0.1/1, 0.2/2, 0.3/3, 0.4/4,\}$$

As a result, we find an optimizing set  $U_{io}$ , which allows you to subtract the best alternative, which means the probable disease corresponding to the symptom complex:

$$A^* = \mu_{A_o}^{\sim}(a_o) = \max \mu_{A_o}^{\sim}(a_i) \quad (5)$$

where  $\mu_{A_o}^{\sim}(a_i) = \max \mu_{u_{io}}^{\sim}(u_k)$

$$U_{1o} = \{0.5/7, 0.8/8, 0.5/9, 0.7/10, 0.2/2, 0.1/1\}$$

$$U_{2o} = \{0.2/2, 0.1/1, 0.2/2, 0.3/3, 0.4/4,\}$$

$$\mu^{\sim}(A1) = \max(0.5, 0.8, 0.5, 0.7, 0.2, 0.1) = 0.8;$$

$$\mu^{\sim}(A2) = \max(0.2, 0.1, 0.2, 0.3, 0.4) = 0.4$$

$$A(*) = \max(0.8, 0.4) = 0.8(A1).$$

Finally, with the existing symptom complex and the severity of symptoms, the most likely disease: Primary biliary cirrhosis of the liver.

There are symptoms where the patient's serum immunoglobulin content is elevated (X1), with a high titer of antimitochondrial antibodies detected in the blood (X2). Antibodies to smooth muscles are also present in the blood (X3). Furthermore, there is an elevated level of alanine aminotransferase in the blood (X4), indicating liver damage. Additionally, the patient exhibits an advanced stage of the disease with portal hypertension syndrome (X5).

By utilizing the table for the differential diagnosis between primary biliary cirrhosis of the liver and active hepatitis with the cholestatic syndrome, we can create a utility matrix to assess these symptoms. The following set represents the patient's state:

$$X = \bigcup_k \mu^{\sim}(Xk) / Xk, \quad (6)$$

where  $x_k \in X$  and  $\mu^{\sim}(x_k)$  the severity of symptom  $k$ .

In practical medicine, predicting the progression of diseases in patients is crucial.

This can be achieved through a mathematical approach. It involves tracking changes in diagnostic results based on the severity of symptoms, which inherently vary throughout observation. Predicting the progression of diseases in patients is a critical aspect of healthcare

that can significantly impact treatment decisions, patient outcomes, and resource allocation. By utilizing various techniques and technologies, healthcare professionals aim to forecast how a disease will advance in an individual, allowing for personalized interventions and improved management strategies [29]. One of the key factors in predicting disease progression is the collection and analysis of relevant patient data. We have obtained the following results presented in Table 4 below.

Table 4: Results of diseases from changes in the degree of expression of symptoms

Nº	Patient's data	Probability of diseases
1	0.9/X1; 0.9/X2; 0.6/X3; 0.5/X4; 0.9/X5	0.99 (A1); 0.5 (A2)
2	0.7/X1; 0.7/X2; 0.6/X3; 0.5/X4; 0.7/X5	0.91 (A1); 0.5 (A2)
3	0.5/X1; 0.5/X2; 0.6/X3; 0.5/X4; 0.5/X5	0.75 (A1); 0.8 (A2)
4	0.3/X1; 0.3/X2; 0.6/X3; 0.5/X4; 0.3/X5	0.51 (A1); 0.8 (A2)
5	0.1/X1; 0.1/X2; 0.6/X3; 0.5/X4; 0.1/X5	0.4 (A1); 0.8 (A2)

Based on the current symptom complex and the severity of symptoms (1.2), primary biliary cirrhosis of the liver is observed with probabilities of 0.99 and 0.91. As the severity of the symptom decreases (3, 4, 5), the probability of active hepatitis with cholestatic syndrome becomes dominant and remains constant at 0.8 (fig.4).

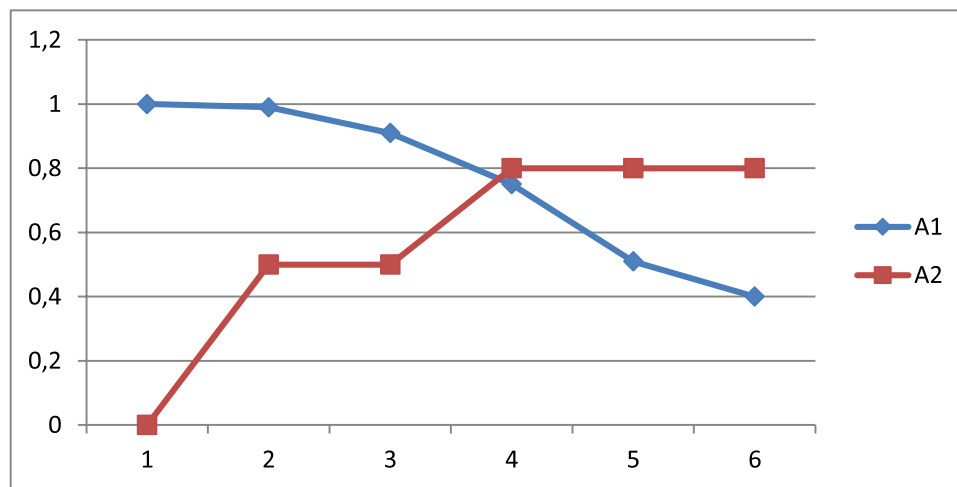


Figure 4: Dependence of the outcome of the disease on time

There is the difference between the probability of these two diseases increases from 0.05 to 0.4 (fig.5). Predicting the progression of diseases in patients is a multifaceted and rapidly evolving field.

## Discussion

Automated diagnostic systems can effectively utilize this mathematical model. Implementing such systems will greatly enhance the professional capabilities of doctors, allowing them to benefit from advanced diagnostic tools and techniques. Through the integration of diverse patient data, advanced machine learning techniques, and longitudinal analyses, healthcare professionals can develop predictive models that assist in individualized treatment planning, early intervention, and resource allocation. Healthcare providers can strive for improved patient outcomes and more efficient healthcare delivery by harnessing the power of data-driven

predictions. Predictive models play a crucial role in this process by leveraging available data and employing algorithms to estimate disease progression with varying degrees of accuracy.

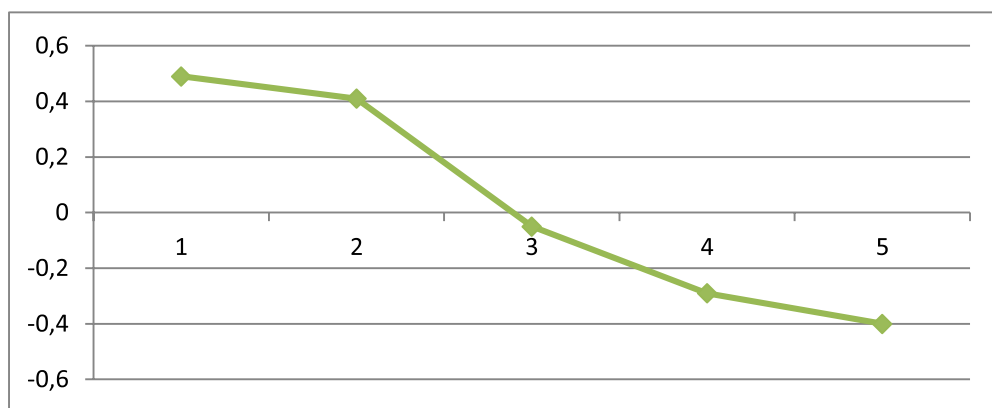


Figure 5: The difference between the probabilities of the diseases

*Limitations and problems on mathematical modeling in diagnosing diseases:*

Machine learning algorithms used in models can inherit biases present in historical data, potentially leading to unfair or discriminatory diagnostic results, especially among underrepresented groups. Some advanced mathematical models, such as deep learning neural networks, can be highly complex and challenging to interpret, making it difficult for healthcare professionals to trust and understand the model's decisions. Moreover, healthcare providers may be resistant to adopting mathematical models due to unfamiliarity or skepticism about their accuracy and utility, leading to slow adoption rates. Also, ensuring that mathematical models remain accurate and relevant over time requires ongoing validation efforts, which can be resource-intensive and time-consuming.

To sum up, while mathematical modeling holds significant promise for improving healthcare diagnostics, there are several limitations and challenges that must be addressed to realize its full potential. These issues range from data quality and privacy concerns to regulatory and ethical considerations, as well as the need for ongoing model validation and adaptation. Successfully integrating mathematical modeling into healthcare workflows requires a multi-disciplinary approach and ongoing collaboration between healthcare professionals and data scientists.

The accuracy of the model directly depends on the level of knowledge on a specific disease, in our case on pyelonephritis, and needs constant updating of the training set of symptoms in connection with updates of clinical studies. Limitations and problems for such a model are the inaccuracy and untruthfulness of the patient, who may inadvertently mislead the model by entering inaccurate data about his condition [30]. The presence of a doctor when filling in the picture of the disease by the patient lowers the level of this problem.

The limitation is also caused by the lack of clinical data that can be used to train the model, since there are very few datasets collected for patients with pyelonephritis.

Validation of the model was also carried out on the data of patients with pyelonephritis [31, 32].

**Conclusion**

Deterministic methods for automating the diagnosis of diseases are based on mathematical models and algorithms that search for a partial match between the symptoms of an observed patient and the symptoms of previously observed patients whose diagnoses are known.

These methods use statistical and other mathematical models to generate a diagnosis based on the available data. However, because they need the ability to form an explanation of the result obtained that is understandable to doctors, their use can be limited.

Logical methods, on the other hand, use formal logic and reasoning to generate a diagnosis. These methods can generate an explanation of the reasoning behind the Diagnosis, which can be more understandable to doctors. However, logical methods can be limited by their reliance on precise and complete data.

Probabilistic methods are based on probability theory and statistical analysis, and they can generate a diagnosis along with a probability estimate. These methods can be useful when dealing with uncertain or incomplete data. However, they can be complex to implement and require a large amount of data to be effective.

Moreover, the choice of method for automating the Diagnosis of diseases depends on the specific application and available data. A combination of methods may be used to improve accuracy and usability. Both logical and probabilistic methods for automating the Diagnosis of diseases aim to make a diagnostic decision based on the knowledge of experts. These methods can construct an explanation component, which can help improve their usability and interpretability.

Medical diagnostic systems are designed to replicate a doctor's decision-making process by processing rules established by experts that link observations to diseases. These systems are a prominent application of artificial intelligence in medicine, but they are not universal and vary widely in their subject area, diagnostic mechanism, and knowledge storage method.

Because medical diagnostic systems are highly specialized, they are not universal and cannot be used to diagnose all diseases. Instead, each system is designed for a specific subject area and uses a unique mechanism for making a diagnosis. The knowledge used by these systems is typically stored in a knowledge base, which can be updated as new information becomes available.

*The model will be integrated* into clinical workflows as an application for diagnosing the patient's condition, where he can select more specific related descriptions of his condition using hints, thereby improving the accuracy of his clinical history. This facilitates the work of the questionnaire for the attending physician, and is also an additional diagnostic tool.

*Further research:* Exploring hybrid models that combine deterministic, logical, and probabilistic methods could harness the strengths of each approach. Developing innovative algorithms that dynamically switch between these methods based on data availability and complexity could lead to more accurate and adaptable diagnostic systems. Future research should focus on developing methods that generate human-understandable explanations for diagnostic decisions, regardless of the underlying algorithm. This would foster trust among medical professionals and patients. Leveraging advancements in machine learning, such as deep learning and reinforcement learning, for disease diagnosis is an exciting avenue. These techniques have shown promise in image analysis and natural language processing, potentially aiding in more accurate and nuanced diagnoses. Tailoring methods to the specific needs of these disciplines can lead to more accurate and specialized diagnostic tools.

The algorithms used by medical diagnostic systems can vary depending on the specific application, but they are generally designed to process the rules and information stored in the knowledge base to generate a diagnosis. These algorithms can be deterministic, logical, or probabilistic and may use machine-learning techniques to improve accuracy over time.

Therefore, medical diagnostic systems are an important application of artificial intelligence in medicine and can help to improve the accuracy and efficiency of diagnoses. However, they are not a replacement for human doctors and must be used with clinical expertise and judgment.

In summary, the future of automated medical diagnosis lies in the integration of diverse approaches, improved data management, ethical considerations, and a continued commitment to enhancing both accuracy and usability. By addressing these research areas, we can pave the way for more effective and accessible healthcare diagnostic systems.

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